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Cross-border shopping and the Atkinson–Stiglitz theorem

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Abstract We introduce cross-border shopping and indirect tax competition into a model of optimal taxation. The Atkinson–Stiglitz result that indirect taxation cannot improve the efficiency of information-constrained tax-transfer policies, and that indirect taxes should not be differentiated across goods, is shown to hold in this case even if countries are asymmetric. However, if the tax system must contain indirect taxation, differentiated indirect tax rates arise in the equilibrium and restricting differentiated indirect taxation can be welfare-increasing.

Keywords Cross-border shopping · Atkinson–Stiglitz theorem · Tax competition · Direct and indirect taxes

JEL Classification H21 · F15

1 Introduction

The completion of the single European Market in the early 1990s removed the barriers to cross-border shopping but left the right to set indirect taxes mostly to the individual Member States. As a consequence, economists have studied tax competition in indirect taxes for cross-border shoppers from a theoretical perspective, cf. Mintz and Tulkens (1986), Kanbur and Keen (1993), Nielsen (2001, 2002), as well as empirically, cf. Lockwood and Migali (2009), Asplund et al. (2007), Devereux et al. (2007), Jacobs et al. (2010), and Agrawal (2011), among others. While these studies have enriched our understanding of indirect tax competition, and support its empirical relevance, they have not shed much light on its consequences for other aspects of the tax system, such as the interplay between direct and indirect taxes.

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The present study considers the implications of cross-border shopping for optimal direct taxation and governments' capabilities to implement incentive-compatible redistributive taxation. More specifically, we investigate the role of cross-border shopping for the validity of the Atkinson–Stiglitz theorem. In their seminal contribution, Atkinson and Stiglitz (1976), henceforth AS, showed that uniform commodity taxation is optimal, if preferences are separable in labor and consumption, and the government implements an optimal non-linear direct taxation scheme. This result is valid for any level of uniform commodity taxation in a closed economy, and is commonly regarded as one of the most policy relevant optimal taxation results. Accordingly, many scholars have scrutinized its scope and robustness. Naito (1999) showed that the result no longer applies, if wages are determined endogenously. Saez (2004), however, demonstrated that it can be restored if human capital formation is also made endogenous. Similarly, Cremer et al. (2001) demonstrated that different wealth endowments result in optimally differentiated indirect taxes, and Boadway and Pestieau (2003) analyzed how other aspects such as differences in needs, different types of labor, or household production can lead to optimally differentiated indirect taxes, even with separable preferences. Finally, Kaplow (2006) and Laroque (2005) have strengthened the case for uniform taxation showing that direct taxes must not necessarily be optimal for the AS result to apply.[1]

The existing literature has typically maintained the closed economy setting. Given the increasing market integration in Europe, and elsewhere, we depart from the closed economy assumption and analyze the implications of cross-border shopping and tax competition for the AS result. Our key research question is, whether the additional constraint of indirect tax competition implies that a government, in its desire to implement an incentive-compatible tax-transfer policy, optimally differentiates indirect taxes between goods which are subject to cross-border shopping and those goods which are not. The analysis incorporates cross-border shopping into the framework of Boadway and Pestieau (2003), which itself is an extension of the Stiglitz's (1982) two-type optimal taxation model. Individuals are tied to their place of residence for work, but are mobile regarding the purchase of certain consumption goods. The approach turns the model into a strategic tax competition framework in which the governments try to redistribute subject to their information constraints, and compete for cross-border shoppers.

We derive a number of results. First, the AS result holds even with competition for cross-border shoppers. Indirect taxes are uniformly set to zero for all goods. Thus, in general, the argument for uniform indirect taxation remains valid. This result is independent of the nature of the competing countries. In particular, asymmetries regarding the relative size of the countries, the governments' relative welfare weights, or skill differences are not affecting its validity. This is contrary to the existing partial equilibrium models of indirect tax competition, such as Kanbur and Keen (1993) or Nielsen (2001), which directly link country asymmetries to equilibrium policies.

[1] Our approach also relates to studies that have introduced aspects of tax competition into optimal taxation models. Huber (1999) considers the interaction between optimal income taxation and the taxation of internationally mobile capital. Simula and Trannoy (2010) and Lipatov and Weichenrieder (2010) introduce labor mobility into the optimal taxation framework and study the resulting implications for the optimal tax schedule.

However, as we also show, the scope of the result of uniform indirect taxes is more limited with cross-border shopping. It is necessarily tied to the abstinence from indirect taxation. If countries rely on indirect taxation for exogenous reasons they will differentiate their indirect taxes systematically. Whether countries choose lower or higher taxes for goods that are subject to cross-border shopping depends on the level of indirect taxes relative to the neighboring countries. Moreover, country characteristics matter with exogenous indirect taxation. With the same level of exogenous indirect taxation and sufficient differences in country size, smaller countries choose lower tax rates, in line with the results of the existing partial equilibrium models.

The paper is organized as follows. Section 2 outlines the framework which we use in Sect. 3 to study the general case without restrictions on indirect taxation. Section 4 considers the restriction, that at least one of the countries uses indirect taxation, and also studies the implications of country asymmetries. Finally, Sect. 5 discusses the results and their policy implications.

2 The framework

There are two neighboring countries $i = s, b$, with a mass of individuals in country s (small) equal to A^s, and equal to A^b in country b (big), $A^b \geq A^s$. In both countries there are low and high productivity individuals $j = l, h$, who only differ in their productivity w^i_j, $w^i_h > w^i_l$, and with their relative shares given by λ^i_j, $\lambda^i_l + \lambda^i_h = 1$. Preferences are represented by the strictly concave utility function $U^i_j = u(g(x^i_j, z^i_j), l^i_j)$, which is separable in the sub-utility of consumption $g(.,.)$ and labor l. Gross income of an individual of productivity j in country i is denoted by y^i_j, $y^i_j = l^i_j w^i_j$. There are two normal consumption goods, x and z, with producer prices set to one and country-specific consumer prices $q^i_k = 1 + t^i_k$, where t^i_k is the specific tax levied on good $k = x, z$, in country i. Transport costs for good x are prohibitively high, so that it can only be bought domestically, whereas good z can be bought in the neighboring country subject to transport costs.[2]

As in Boadway and Pestieau (2003), the individual utility maximization problem for any given tax policy may be broken up into two stages due to separability between labor and all other consumption goods. At the first stage, individuals choose labor supply l^i_j, which determines net income c^i_j, given the income tax schedule. At the second stage individuals choose the consumption of x^i_j and z^i_j, and the quantity of good z bought abroad, denoted by $z^{i,-i}_j$.

The cross-border shopping decision is modeled following Haufler (1996). Transport costs for good z are quadratic in the volume of cross-border shopping. Total transportation costs are $K(z^{i,-i}_j) = \frac{a^i}{2}(z^{i,-i}_j)^2$, such that the relevant marginal transport costs are $a^i z^{i,-i}_j$. We assume that these costs depend positively on country size in

[2]This dichotomy serves as a benchmark. Although for most practical purposes transport costs may be prohibitively high for many goods and services, conceptually, most goods, even including those with high transport cost, may be, in principle, subject to cross-border shopping. Note also that wholesale transport costs are assumed to be zero.

the simple linear form $a^i = aA^i$, $a > 0$. The interpretation is that, on average, people in the large country are further away from the border and therefore have larger transport costs, on average. This assumption has the economically intuitive consequence that the marginal effects of reducing t_z^i or t_z^{-i} on the respective indirect tax bases are the same for both countries.[3]

Buying one more unit abroad will be preferred to shopping at home if the price abroad plus the marginal transport costs is less than the price in the country of residence, i.e., if $t_z^{-i} + a^i z_j^{i,-i} < t_z^i$. Note that the decision about whether, and how much, to buy abroad is the same for high and low productivity individuals, because transport costs are modeled as monetary costs, and not as time costs, which might differ between the individuals. Consequently, for given tax rates t_z^i and t_z^{-i}, consumers are indifferent between buying at home or abroad at quantity

$$\left(z_j^{i,-i}\right)^* = \frac{t_z^i - t_z^{-i}}{a^i}. \tag{1}$$

Since cross-border shopping is restricted to positive amounts, we have $z_j^{i,-i} \equiv \max[0, (z_j^{i,-i})^*]$. By assumption, individuals always buy some amount of good z in their home country, such that $z_j^{i,-i} < z_j^i$. We discuss the implications of different transport costs for different individuals in Sect. 5.

We focus on the welfare maximization problem of government i, noting that the other government $-i$ solves an analogous problem. The government can observe gross incomes as well as anonymous transactions in the market. Its tax policy instruments are a non-linear income tax and indirect taxes, and we reformulate utility in terms of variables that are observable by the government as $v_j^i(f(q_z^i, q_z^{-i}, c_j^i), \frac{y_j^i}{w_j^i}) \equiv u(f(q_z^i, q_z^{-i}, c_j^i), l_j^i)$, where $f(.)$ denotes the value function of the second stage of the individual utility maximization problem. The government maximizes a utilitarian welfare function with welfare weights α_j^i:

$$W^i = \sum_{j=h,l} \alpha_j^i A^i \lambda_j^i v_j^i \left(f\left(q_z^i, q_z^{-i}, c_j^i\right), \frac{y_j^i}{w_j^i} \right). \tag{2}$$

The government's budget constraint is

$$\begin{aligned} 0 \leq & \sum_{j=h,l} \lambda_j^i \left(y_j^i - c_j^i\right) A^i + \sum_{j=h,l} \lambda_j^i A^i t_z^i \left[z_j^i \left(q_z^i, q_z^{-i}, c_j^i\right) - z_j^{i,-i}\left(q_z^i, q_z^{-i}\right)\right] \\ & + \sum_{j=h,l} A^{-i} \lambda_j^{-i} t_z^i z_j^{-i,i}\left(q_z^i, q_z^{-i}\right) + \sum_{j=h,l} \lambda_j^i A^i t_x^i x_j^i \left(q_z^i, q_z^{-i}, c_j^i\right) \equiv B. \end{aligned} \tag{3}$$

The first term of the government budget consists of the direct taxes or subsidies paid by, or to, the low productivity and high productivity individuals, respectively. The

[3] In a potential spatial interpretation, this corresponds to the analysis of Nielsen (2001), where countries differ in size but have the same population density, which also results in equal marginal effects of tax reductions on the respective tax bases for both countries.

second term is the tax revenues from the domestic purchases of good z. The third term adds the taxes paid by the cross-border shoppers from the neighboring country. The last term is the revenues from the indirect taxes on good x.

The government's tax policy also has to be incentive-compatible. We consider downward incentive compatibility only. This condition is

$$v_h^i\left(f\left(q_z^i, q_z^{-i}, c_h^i\right), \frac{y_h^i}{w_h^i}\right) \geq v_h^i\left(f\left(q_z^i, q_z^{-i}, c_l^i\right), \frac{y_l^i}{w_h^i}\right). \tag{4}$$

By assumption, the share of low productivity types is sufficiently large such that the optimal policy entails positive labor supply from both types.

The government's problem is to maximize (2) subject to (3) and (4) choosing $y_l^i, y_h^i, c_l^i, c_h^i$ and q_z^i.[4] From the resulting first-order conditions for c_l^i, c_h^i and q_z^i we derive after further manipulation (see the Appendix for details),

$$0 = \sum_{j=h,l} A^i \lambda_j^i t_z^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i A^i t_x^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} - \frac{t_z^i}{a} + \sum_{j=h,l} A^{-i} \lambda_j^{-i} z_j^{-i,i} \equiv F^i, \tag{5}$$

where $\frac{\partial \tilde{z}_j^i}{\partial q_z^i}$ and $\frac{\partial \tilde{x}_j^i}{\partial q_z^i}$ are the derivatives of the compensated demand functions with respect to prices. This expression F^i is at the heart of our further discussion of the potential equilibria. Note that for country $-i$ an analogous expression F^{-i} must hold in the equilibrium.

With separable preferences, incentive-compatible non-linear income taxation and linear indirect taxation our framework has the key assumptions that are necessary to deliver the AS result, but we additionally introduce cross-border shopping. Restricting $z_j^{i,-i}$ and $z_j^{-i,i}$ to equal zero reduces the analysis to the closed economy benchmark studied by Boadway and Pestieau (2003, p. 390). As they show, optimality requires $t_z^i = t_x^i$ in this case, in line with the original AS result. In the remainder, we study whether with cross-border shopping the optimal policy still requires undifferentiated indirect taxes.

3 The unrestricted equilibrium

Consider first the general case of asymmetric countries without any a priori restrictions on indirect taxes. In line with the closed economy literature, we normalize the tax on good x to zero in both countries. In the general asymmetric case we have the following result:

Proposition 1 *In an equilibrium with tax rates $t_x^i = t_x^{-i} = 0$, both countries set $t_z^i = t_z^{-i} = 0$.*

[4] There is no first-order condition for q_x^i, since t_x^i will always be set exogenously.

Proof See Appendix. □

The intuition for this equilibrium is as follows. Given that the neighboring country sets $t_z^{-i} = 0$, country i has nothing to gain from lowering its tax t_z^i below zero. This attracts foreigners to cross-border shop, and the government effectively subsidizes them given that $t_z^i < 0$. This must decrease domestic welfare. Moreover, distorting prices between x and z has an additional negative affect on welfare. This follows from the general logic of the original AS result. Alternatively, the government also has nothing to gain from increasing t_z^i above zero. This creates outbound cross-border shopping and a loss of revenue. Moreover, it also distorts prices. The optimal policy is therefore not to differentiate indirect taxation and to set the tax for goods that are subject to cross-border shopping to zero. Note that this intuition does not depend on country characteristics, in line with the result being independent of potential asymmetries between countries.

Thus, the baseline AS result holds in the general case of open economies with cross-border shopping. Despite the possibility to attract cross-border shoppers, indirect taxes cannot increase the efficiency of the tax system, and tax rates should be equalized across goods. This result is valid independently of relative country size, population shares of high and low skilled individuals or the utility weights assigned to them. This is in contrast to the partial equilibrium models of cross-border shopping, such as Kanbur and Keen (1993) and Nielsen (2001), where size differences determine tax rates and flows of cross-border shoppers in the equilibrium.

4 Equilibria with indirect taxation

Our analysis has so far set the tax rate on goods that are not subject to cross-border shopping, t_x, to zero in both countries. In the closed economy situation, setting $t_x > 0$ is irrelevant for the AS result, and $t_x = t_z$ can be derived as the optimal tax policy, since this is just equivalent to the corresponding increases in income taxes. We now investigate, whether this remains the case with cross-border shopping. This is an important consideration since, in practice, countries typically rely on a mix of direct and indirect taxes. As argued by Boadway et al. (1994), and others, such a tax mix can be motivated by tax administration and/or tax evasion reasons.

We first study the case where good x is taxed at the same exogenous rate in both countries, i.e., $t_x^i = t_x^{-i} = t_x > 0$. We have the following proposition:

Proposition 2 (i) *If $t_x^i = t_x^{-i} = t_x > 0$, the countries choose $t_z^i < t_x$ and $t_z^{-i} < t_x$ in equilibrium, irrespective of their characteristics.* (ii) *If the two countries are symmetric, the symmetric equilibrium is characterized by $t_x^i = t_x^{-i} = t_x > t_z^i = t_z^{-i} = t_z > 0$, and $t_z \to t_x$, if $a \to \infty$.*

Proof See Appendix. □

Requiring the additional restriction $t_x > 0$ changes the situation. The AS result no longer holds. Both countries have an incentive to differentiate indirect tax rates to attract cross-border shoppers. The optimal policy trades off the distortion due to differentiated indirect taxes with the competitive pressures originating from the competition for cross-border shoppers. The optimal solution to this trade-off is a positive, but lower tax rate on z. The trade-off does not arise when t_x is fixed at zero in both countries, because $t_z^i = 0$ then simultaneously avoids losses from distorted consumer prices, and from cross-border shopping. As is evident from the result for symmetric countries, the degree of tax rate differentiation depends on the intensity of tax competition. As transport costs become very high ($a \to \infty$) rate differentiation vanishes. Proposition 2 also has welfare implications:

Corollary 3 *For symmetric countries with the same exogenous level of indirect taxes on goods that are not subject to cross-border shopping, welfare is lower for both countries in the equilibrium than in the global, information-constrained second best.*

Proof An information-constrained social planner maximizing the joint welfare of both countries would set $t_z = t_x$ in both countries. This follows directly from the AS theorem. Thus, in the information-constrained second best there are no cross-border shoppers. In the non-cooperative symmetric equilibrium there are also no cross-border shoppers but indirect taxes are differentiated, implying a lower level of welfare in both countries. □

If indirect taxation plays an important role in the countries under consideration, tax competition for cross-border shopping has the potential to reduce welfare, since it induces tax differentiation, which generates distortions without improving redistribution. As follows directly from Corollary 3 for symmetric countries, welfare can be increased if these countries were able to agree not to differentiate indirect taxes.

Consider now a constellation where the two countries differ in size but are identical in all other aspects including the same exogenous tax rate on the good that is not subject to cross-border shopping. The structure of indirect taxation can be further characterized in this case:

Proposition 4 *If the two countries only differ in size, and this difference is sufficiently large, the big country does not tax the good that is subject to cross-border shopping at a lower rate than the small country.*

Proof See Appendix. □

This proposition is in line with the results of the literature on commodity tax competition. Smaller countries have a smaller indirect tax base to serve as a counterweight to the incentive to reduce taxes to attract marginal cross-border shoppers. Due to its smaller population, distorting relative prices is less costly relative to the gains from additional revenues due to inward cross-border shopping for the small country.

To complete our discussion of equilibria with indirect taxation, we address the case where only one country has an exogenous positive tax rate on good x, i.e., $t_x^i > t_x^{-i} = 0$. Such differences may be due to different innate country characteristics, such as the reliance on indirect taxes as an instrument to address tax evasion. Alternatively, this may reflect different standard VAT rates which we observe in different countries. We have our next result:

Proposition 5 *If* $t_x^i > t_x^{-i} = 0$, *an equilibrium must entail* $t_x^i > t_z^i > t_z^{-i} > 0$.

Proof See Appendix. □

Indirect taxation in one country triggers indirect taxation in the other country, even if this country would not rely on indirect taxes in the absence of cross-border shopping. Rates are differentiated in both countries but the pattern of differentiation is different from the case of symmetric exogenous taxes. The high tax country uses lower taxes on z to reduce the loss of tax revenue due to outbound cross-border shopping. The tax on good z in the high tax country offers the low tax country a possibility to generate additional revenues from inward cross-border shopping by choosing a lower but positive rate. Note that, in this case, forcing countries to use uniform rates at $t_z^i = t_x^i$ and $t_z^{-i} = 0$ would not necessarily make both of them better off. While both countries would benefit from undistorted consumption, as indicated by the original AS logic, country i would suffer a greater outflow of cross-border shopping with the associated loss in revenues due to the larger tax differential. But country $-i$ would also lose indirect tax revenue since the cross-border shoppers would no longer be paying any tax in country $-i$.

5 Discussion and conclusion

Our analysis has shown that, in general, the AS result remains valid in an open economy setting with tax competition for cross-border shoppers. Indirect taxation cannot increase the efficiency of the tax system. However, in the open economy, this result is no longer equivalent to uniform taxation at any positive rate. Tax rates only remain non-differentiated, if goods that are not subject to cross-border shopping can remain untaxed. If tax systems must rely on indirect taxes, the equilibrium will entail tax rate differentiation.

Since we have used a simple framework to allow a straightforward comparison with the closed economy benchmark, a discussion of some of our assumptions is in order. First, our analysis has used identical transport costs. With different transport costs for high and low productivity individuals, these buy different quantities as cross-border shoppers, and also the quantities bought in the domestic market by low productivity individuals and potential mimickers differ in the high tax country. This allows to relax incentive compatibility by differentiating indirect taxes. It is straightforward to derive an analogous expression to (5) for the high tax country, showing that it should additionally increase (decrease) indirect taxation of the good that is subject to cross-border shopping if high productivity individuals have higher (lower) transport costs.[5] Second, one may also consider the case of revenue-maximizing gov-

[5] However, since the option to differentiate indirect taxes to ease incentive compatibility is only available to the high tax country, the best responses are not necessarily continuous. Either country may have an incentive to become the high tax country, such that, in general, the analogous expression to (5) not necessarily characterizes the optimal policy.

ernments. The results, however, are very similar in this case. Without an exogenous restriction on indirect taxes, indirect tax rates will not be differentiated and be set to zero. With an exogenous restriction to use positive rates, differentiation will occur analogously to our analysis. Finally, we assumed that individuals are completely immobile regarding their place of work. In reality, mobility of workers also plays a role for the optimal design of tax systems. Our approach is therefore complementary to studies that focus on the mobility of workers for the design of optimal direct taxation only, but do not take mobile consumers into account.

Our results also have policy implications. An important question regarding the European VAT system is the extent to which Member States should be allowed to apply lower VAT rates, i.e., the range of products and services to which reduced rates may be applicable, as well as the available rate reductions. Given the importance of indirect taxes in Europe, our analysis suggests potential welfare gains from limiting tax differentiation for goods and services that are suited for cross-border shopping. Such limitations are the more likely to be welfare-improving the more standard rates are aligned.

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Appendix: Derivation of key expression (5)

Consider the problem of maximizing (2) s.t. (3) and (4). The Lagrangian is

$$\mathcal{L} = \sum_{j=h,l} \alpha_j^i A^i \lambda_j^i v_j^i \left(f\left(q_z^i, q_z^{-i}, c_j^i\right), \frac{y_j^i}{w_j^i} \right) + \mu B + \gamma \left[v_h^i - \hat{v}_h^i \right],$$

where $v_h^i = v_h^i(f(q_z^i, q_z^{-i}, c_h^i), \frac{y_h^i}{w_h^i})$, and $\hat{v}_h^i = v_h^i(f(q_z^i, q_z^{-i}, c_l^i), \frac{y_l^i}{w_h^i})$ denotes the utility of a mimicker. The government budget constraint can be simplified using

$$\sum_{j=h,l} A^{-i} \lambda_j^{-i} t_z^i z_j^{-i,i} - \sum_{j=h,l} \lambda_j^i A^i t_z^i z_j^{i,-i} = \frac{t_z^i (t_z^{-i} - t_z^i)}{a}.$$

We leave out the first-order conditions with respect to y_l^i, y_h^i, μ and γ and focus on those with respect to c_l^i, c_h^i and q_z^i:

$$c_l^i: \quad \alpha_l^i A^i \lambda_l^i \frac{\partial v_l^i}{\partial f} \frac{\partial f}{\partial c_l^i} - \mu \lambda_l^i A^i \left[1 - t_z^i \frac{\partial z_l^i}{\partial c_l^i} - t_x^i \frac{\partial x_l^i}{\partial c_l^i} \right] - \gamma \frac{\partial \hat{v}_h^i}{\partial f} \frac{\partial f}{\partial c_l^i} = 0, \tag{6}$$

$$c_h^i: \quad \alpha_h^i A^i \lambda_h^i \frac{\partial v_h^i}{\partial f} \frac{\partial f}{\partial c_h^i} - \mu \lambda_h^i A^i \left[1 - t_z^i \frac{\partial z_h^i}{\partial c_h^i} - t_x^i \frac{\partial x_h^i}{\partial c_h^i} \right] + \gamma \frac{\partial v_h^i}{\partial f} \frac{\partial f}{\partial c_h^i} = 0, \tag{7}$$

$$q_z^i: \sum_{j=h,l} \alpha_j^i A^i \lambda_j^i \frac{\partial v_j^i}{\partial f} \frac{\partial f}{\partial q_z^i} + \mu \sum_{j=h,l} A^{-i} \lambda_j^{-i} z_j^{-i,i} + \gamma \left[\frac{\partial v_h^i}{\partial f} \frac{\partial f}{\partial q_z^i} - \frac{\partial \hat{v}_h^i}{\partial f} \frac{\partial f}{\partial q_z^i} \right]$$
$$- \mu t_z^i \frac{1}{a} + \mu A^i \left[t_z^i \sum_{j=h,l} \lambda_j^i \frac{\partial z_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i \left(z_j^i - z_j^{i,-i} \right) + t_x^i \sum_{j=h,l} \lambda_j^i \frac{\partial x_j^i}{\partial q_z^i} \right] = 0. \tag{8}$$

These conditions implicitly define the best response and must be, together with the counterparts for country $-i$, fulfilled in equilibrium. We multiply (6) by $z_l^{i,i}$ and (7) by $z_h^{i,i}$ and add them to (8) to find

$$\sum_{j=h,l} \alpha_j^i A^i \lambda_j^i \frac{\partial v_j^i}{\partial f} \frac{\partial f}{\partial q_z^i} + \sum_{j=h,l} \alpha_j^i A^i \lambda_j^i \frac{\partial v_j^i}{\partial f} \frac{\partial f}{\partial c_j^i} z_j^{i,i} + \mu A^i t_z^i \sum_{j=h,l} \lambda_j^i \frac{\partial z_j^i}{\partial q_z^i}$$
$$+ \mu A^i t_z^i \sum_{j=h,l} \lambda_j^i \frac{\partial z_j^i}{\partial c_j^i} z_j^{i,i} + \mu A^i \sum_{j=h,l} \lambda_j^i z_j^i - \mu A^i \sum_{j=h,l} \lambda_j^i z_j^{i,-i} - \mu A^i \sum_{j=h,l} \lambda_j^i z_j^{i,i}$$
$$+ \mu A^i t_x^i \sum_{j=h,l} \lambda_j^i \frac{\partial x_j^i}{\partial q_z^i} + \mu A^i t_x^i \sum_{j=h,l} \lambda_j^i \frac{\partial x_j^i}{\partial c_j^i} z_j^{i,i} + \mu \sum_{j=h,l} A^{-i} \lambda_j^{-i} z_j^{-i,i} - \mu t_z^i \frac{1}{a}$$
$$+ \gamma \frac{\partial v_h^i}{\partial f} \frac{\partial f}{\partial q_z^i} - \gamma \frac{\partial \hat{v}_h^i}{\partial f} \frac{\partial f}{\partial q_z^i} - \gamma \frac{\partial \hat{v}_h^i}{\partial f} \frac{\partial f}{\partial c_l^i} z_l^{i,i} + \gamma \frac{\partial v_h^i}{\partial f} \frac{\partial f}{\partial c_h^i} z_h^{i,i} = 0.$$

By Roy's Identity the first two terms in the first line add to zero. The last three terms in the second line also equal zero. Given identical transport costs high and low productivity individuals buy the same quantity abroad, and low productivity individuals and mimickers consume the same quantities of good z such that the last line equals zero. Using the Slutsky equation for z and x, this leads to (5).

Proof of Proposition 1 First, with $t_x^i = 0$ the second term of F^i in (5) disappears. Next, $t_z^i = t_z^{-i} = 0$ is compatible with (5) and the corresponding expression F^{-i}. We now show that no other combination of tax rates can be an equilibrium. If $t_z^i < 0$ in (5), then $F^i > 0$, contradicting (5). The same follows from F^{-i} for $t_z^{-i} < 0$. Consider now $t_z^i > t_z^{-i}$, such that $z_j^{-i,i} = 0$. This implies

$$F^i = t_z^i \left(\sum_{j=h,l} A^i \lambda_j^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} - \frac{1}{a} \right) = 0, \tag{9}$$

which is only fulfilled for $t_z^i = 0$ and therefore $t_z^{-i} < 0$. Since neither country subsidizes z, an equilibrium with $t_z^i > t_z^{-i}$ can be ruled out. The combination with t_z^i and t_z^{-i} interchanged is analogous, such that an equilibrium with $t_z^i < t_z^{-i}$ cannot exist. Finally, $t_z^i = t_z^{-i} > 0$ can be ruled out, since it implies (9), which requires $t_z^i = 0$. Thus, only $t_z^i = t_z^{-i} = 0$ is compatible with an equilibrium. □

Proof of Proposition 2 (i) Consider the case $t_z^{-i} \leq t_z^i$. Expression (5) for country i reduces to

$$0 = \sum_{j=h,l} \lambda_j^i \tau_z^i q_z^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i \tau_x^i q_x^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} - \frac{\tau_z^i q_z^i}{a^i}, \tag{10}$$

where $t_k^i \equiv \tau_k^i q_k^i$. Consider now the Hicksian demands and their properties. We know that $\sum_{j=h,l} \lambda_j^i q_z^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i q_x^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} = 0$. Multiplying this expression by τ_z^i, using (10), and rearranging yields

$$\sum_{j=h,l} \lambda_j^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} = \frac{\tau_z^i q_z^i}{a(\tau_x^i - \tau_z^i) q_x^i} \frac{1}{A^i}. \tag{11}$$

This implies $\tau_z^i < \tau_x^i$ and therefore $t_z^i < t_x^i$, since $\sum_{j=h,l} \frac{\partial \tilde{x}_j^i}{\partial q_z^i} > 0$. Since $t_x^i = t_x^{-i} = t_x > 0$ and $t_z^{-i} \leq t_z^i$, country $-i$ chooses a lower tax for good z as well. (ii) For symmetric countries we have $A^i = A^{-i} = A$, $\alpha_j^i = \alpha_j^{-i} = \alpha_j$, and $\lambda_j^i = \lambda_j^{-i} = \lambda_j$. In a symmetric equilibrium, we have $z_j^{-i,i} = 0$. From (5) we get $\sum_{j=h,l} A\lambda_j t_z \frac{\partial \tilde{z}_j^i}{\partial q_z} + \sum_{j=h,l} A\lambda_j t_x \frac{\partial \tilde{x}_j^i}{\partial q_z} - \frac{t_z}{a} = 0$, which requires $t_z > 0$ and corresponds to (10). Following the argumentation in part (i), we therefore have $t_x > t_z$. For $a \to \infty$, $\frac{t_z}{a} \to 0$, and therefore $t_z \to t_x$. □

Proof of Proposition 4 Consider the case $t_x^b = t_x^s = t_x > 0$, $A^b > A^s$ and $\lambda_j^i = \lambda_j^{-i} = \lambda_j$. Assume for contradiction that in equilibrium $t_z^b < t_z^s$. Expression (5) for the big country is

$$0 = \sum_{j=h,l} \lambda_j \tau_z^b q_z^b \frac{\partial \tilde{z}_j^b}{\partial q_z^b} + \sum_{j=h,l} \lambda_j \tau_x^b q_x^b \frac{\partial \tilde{x}_j^b}{\partial q_z^b} + \sum_{j=h,l} \lambda_j \frac{1}{A^b} \frac{\tau_z^s q_z^s - 2\tau_z^b q_z^b}{a}, \tag{12}$$

where again $t_k^i \equiv \tau_k^i q_k^i$. The properties of the Hicksian demands imply $\sum_{j=h,l} \lambda_j \times q_z^b \frac{\partial \tilde{z}_j^b}{\partial q_z^b} + \sum_{j=h,l} \lambda_j q_x^b \frac{\partial \tilde{x}_j^b}{\partial q_z^b} = 0$. Multiplying this with τ_z^b and combining it with (12) yields

$$\sum_{j=h,l} \lambda_j \frac{\partial \tilde{x}_j^b}{\partial q_z^b} = \frac{(2\tau_z^b q_z^b - \tau_z^s q_z^s)}{a(\tau_x^b - \tau_z^b) q_x^b} \frac{1}{A^b}. \tag{13}$$

For the small country as the high tax country, we can use (11) and get

$$\sum_{j=h,l} \lambda_j \frac{\partial \tilde{x}_j^s}{\partial q_z^s} = \frac{\tau_z^s q_z^s}{a(\tau_x^s - \tau_z^s) q_x^s} \frac{1}{A^s}. \tag{14}$$

By (14), $\tau_x^s - \tau_z^s = \Delta > 0$, where Δ is a constant. By (13), as $\frac{1}{A^b} \to 0$, $\tau_z^b \to \tau_x^b$, since $\sum_{j=h,l} \lambda_j \frac{\partial \tilde{x}_j^b}{\partial q_z^b} > 0$. For $\frac{1}{A^b}$ sufficiently small, this implies $\tau_x^b - \tau_z^b < \Delta$. Since $\tau_x^b = \tau_x^s$, this also implies $\tau_z^b > \tau_z^s$. But this contradicts $t_z^b < t_z^s$. □

Proof of Proposition 5 If $t_x^i > 0$ and $t_x^{-i} = 0$, (5) now differs between country i and country $-i$,

$$F^i = \sum_{j=h,l} A^i \lambda_j^i t_z^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i A^i t_x^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} - \frac{t_z^i}{a} + \sum_{j=h,l} A^{-i} \lambda_j^{-i} z_j^{-i,i} = 0, \quad (15)$$

$$F^{-i} = \sum_{j=h,l} A^{-i} \lambda_j^{-i} t_z^{-i} \frac{\partial \tilde{z}_j^{-i}}{\partial q_z^{-i}} - \frac{t_z^{-i}}{a} + \sum_{j=h,l} A^i \lambda_j^i z_j^{i,-i} = 0. \quad (16)$$

The following constellations are potential equilibria: (i) $t_z^{-i} \geq t_z^i = 0$, (ii) $t_z^i = t_z^{-i} > 0$, (iii) $t_z^i > t_z^{-i} = 0$, (iv) $t_z^{-i} > t_z^i > 0$, (v) $t_z^i < 0$, or $t_z^{-i} < 0$, (vi) $t_z^i > t_z^{-i} > 0$. Substituting these possibilities into (15) and (16) shows that only (vi) is compatible with the equilibrium. This constellation $t_z^i > t_z^{-i} > 0$ reduces (15) to $\sum_{j=h,l} A^i \lambda_j^i t_z^i \frac{\partial \tilde{z}_j^i}{\partial q_z^i} + \sum_{j=h,l} \lambda_j^i A^i t_x^i \frac{\partial \tilde{x}_j^i}{\partial q_z^i} - \frac{t_z^i}{a} = 0$, implying $t_x^i > t_z^i$. □

References

Agrawal, D. (2011). *The tax gradient: do local sales taxes reduce tax differentials at state borders?* Paper presented at the Congress of the International Institute of Public Finance, 9 August 2011, Ann Arbor.

Asplund, M., Friberg, R., & Wilander, F. (2007). Demand and distance: evidence on cross-border shopping. *Journal of Public Economics*, *91*(1–2), 141–157.

Atkinson, A. B., & Stiglitz, J. E. (1976). The design of tax structure: direct versus indirect taxation. *Journal of Public Economics*, *6*, 55–75.

Boadway, R., Marchand, M., & Pestieau, P. (1994). Towards a theory of the direct–indirect tax mix. *Journal of Public Economics*, *55*, 71–88.

Boadway, R., & Pestieau, P. (2003). Indirect taxation and redistribution: the scope of the Atkinson–Stiglitz theorem. In R. Arnott, B. Greenwald, R. Kanbur, & B. Nalebuff (Eds.), *Economics for an imperfect world: essays in honor of Joseph E. Stiglitz*. Cambridge: MIT Press.

Cremer, H., Pestieau, P., & Rochet, J.-C. (2001). Direct versus indirect taxation: the design of the tax structure revisited. *International Economic Review*, *42*, 781–799.

Devereux, M. P., Lockwood, B., & Redoano, M. (2007). Horizontal and vertical indirect tax competition: theory and some evidence from the USA. *Journal of Public Economics*, *91*, 451–479.

Haufler, A. (1996). Tax coordination with different preferences for public goods: conflict or harmony of interest? *International Tax and Public Finance*, *3*, 5–28.

Huber, B. (1999). Tax competition and tax coordination in an optimum income tax model. *Journal of Public Economics*, *71*(3), 441–458.

Jacobs, J., Ligthart, J., & Vrijburg, H. (2010). Consumption tax competition among governments: evidence from the United States. *International Tax and Public Finance*, *17*, 271–294.

Kanbur, R., & Keen, M. (1993). Jeux sans frontieres: tax competition and tax coordination when countries differ in size. *The American Economic Review*, *83*, 877–892.

Kaplow, L. (2006). On the undesirability of commodity taxation even when income taxation is not optimal. *Journal of Public Economics*, *90*(6–7), 1235–1250.

Laroque, G. R. (2005). Indirect taxation is superfluous under separability and taste homogeneity: a simple proof. *Economics Letters*, *87*, 141–144.
Lipatov, V., & Weichenrieder, A. (2010). *Optimal income taxation with tax competition*. CESifo working paper 3108.
Lockwood, B., & Migali, G. (2009). Did the single market cause competition in excise taxes? Evidence from EU countries. *Economic Journal*, *119*, 406–429.
Mintz, J., & Tulkens, H. (1986). Commodity tax competition between member states of a federation: equilibrium and efficiency. *Journal of Public Economics*, *29*, 133–172.
Naito, H. (1999). Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency. *Journal of Public Economics*, *71*, 165–188.
Nielsen, S. B. (2001). A simple model of commodity taxation and cross-border shopping. *Scandinavian Journal of Economics*, *103*(4), 599–623.
Nielsen, S. B. (2002). Cross-border shopping from small to large countries. *Economics Letters*, *77*, 309–313.
Saez, E. (2004). Direct or indirect tax instruments for redistribution: short-run versus long-run. *Journal of Public Economics*, *88*, 503–518.
Simula, L., & Trannoy, A. (2010). Optimal income tax under the threat of migration by top-income earners. *Journal of Public Economics*, *94*(1–2), 163–173.
Stiglitz, J. E. (1982). Self-selection and Pareto efficient taxation. *Journal of Public Economics*, *17*, 213–240.